The Measurement and Trend of Inequality: A Basic Revision

By MORTON PAGLIN*

The Lorenz curve has for some sixty years dominated our approach to the measurement of income inequality, and in consequence has become one of the most widely disseminated concepts in economics. This paper will attempt to show that a profound distortion of the degree of income inequality has resulted from the use of the Lorenz curve and the related Gini concentration ratio, both of which combine and hence confuse *intra*family variation of income over the life cycle with the more pertinent concept of *inter*family income variation which underlies our idea of inequality and economic class. To remedy the defects of the Lorenz-Gini approach, a fundamental revision is proposed which takes as its point of departure the replacement of the 45 degree line of equality with a new function generated on the basis of a more careful and explicit definition of perfect equality. This will allow us to utilize annual income data while avoiding the pitfalls now associated with its use. The widely accepted estimates of current inequality, and of historical changes in Gini ratios, are shown to have misrepresented an important characteristic of the *U.S.* economy.

I. Alternatives to Lorenzian Equality

The principal weakness of the Lorenz concept lies buried in the implications of the line of perfect equality. While a few writers have remarked that the 45 degree line has only mathematical significance, they have along with other users of the concept thrust upon it a considerable normative burden. There is really no way to avoid this if the Lorenzian area of inequality is to have any meaning as an index of income distribution. The basic fault with the existing line of equality is that it overspecifies the conditions of equality when used with annual income data. Assuming for the moment no economic growth, the maintenance of Lorenzian equality requires not only equal lifetime incomes, but additionally that families during the period of child rearing, when they have maximum income needs, must have the same incomes as families in the retirement stage of the life cycle when they have minimum economic responsibilities and maximum assets. These conditions are specified by the 45 degree line because families of all ages must have equal incomes in any given year, and this can be realized only if all families have perfectly flat age-income profiles. Yet it would be difficult to argue that a flat age-income profile is essential to equality, and there are substantial reasons for maintaining that such a mechanical requirement is itself highly inequitable. In Figure 1A, average *U.S.* family income by age of family head is indicated by curve *AB* and compared with the flat mean income profile *CD* specified by the Lorenzian equality line. It is certainly reasonable and sufficient to define perfect equality as a condition in which all families have equal lifetime in-

* Professor, department of economics and Graduate Faculty of Urban Studies, Portland State University.
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comes but traverse path AB rather than CD. In fact, to insist on flat age-income profiles flies in the face of both consumption needs and the realities of income production, for we know that investment in human resources through education and training produces a more peaked age-income profile, even apart from lifetime income differences (note Figure 2A). Lorenzian equality also does not allow family income to be increased by additional members entering the labor force. Few economists would explicitly postulate the above conditions as the requirements of perfect equality, yet this is the normative reference level embodied in the Lorenz definition. It is the standard by which we measure inequality and thereby subjectively judge the equity of our economic system.

Some of the limitations of the Lorenz-Gini measure have been recognized in the literature, but the remedies suggested have taken us on a detour of data manipulation: the use of hypothetical lifetime incomes, age-group Gini coefficients, etc. However, none of these approaches has had much impact on our empirical estimates of the trend of inequality, for reasons to be discussed below. Apparently, no one has conceived of the alternative approach: reconstructing the reference line of equality to match the excellent annual income data at our disposal.

Once we cast aside the socially unrealistic 45 degree line of equality, we are free to generate new reference lines corresponding to explicit and reasonable definitions of equality, equity, or Pareto optimality. These new standards will be called P-reference lines, and in the balance of this paper one particular definition will be emphasized and used to make new estimates of inequality in the period 1947 to 1972. The new P-reference line is defined in a way which conforms to what casual users of the L curve might infer is the meaning of equality: equal lifetime incomes but not the added constraint of a

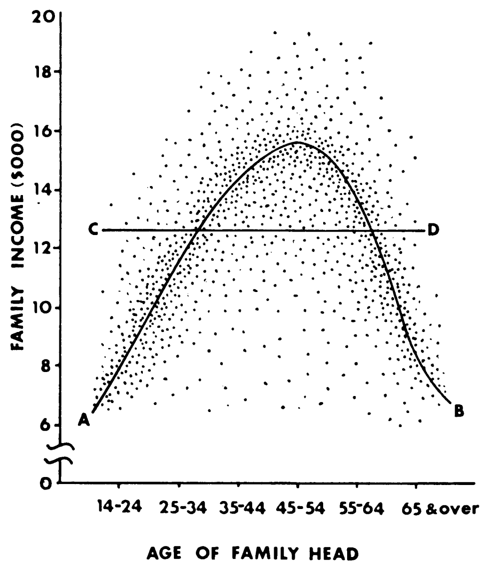

FIGURE 1A. 1972 AVERAGE AGE-INCOME PROFILE (Dots are only suggestive of distribution)

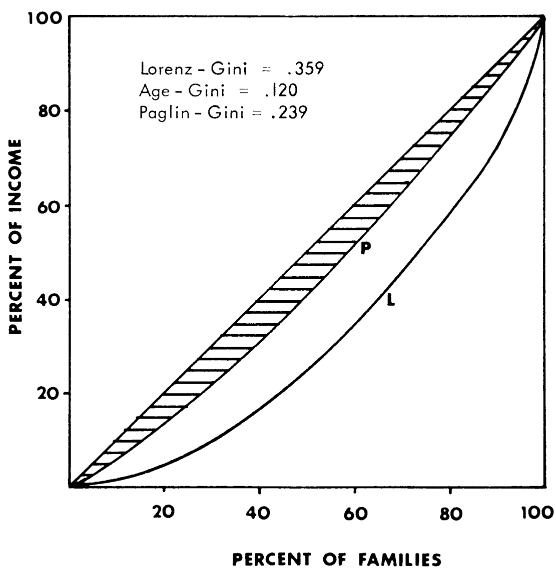

FIGURE 1B. 1972 CUMULATIVE INCOMF DISTRIBUTION SHOWING AGE-RELATED INEQUALITY

Source for both figures: CPR, no. 90, pp. 51–53.

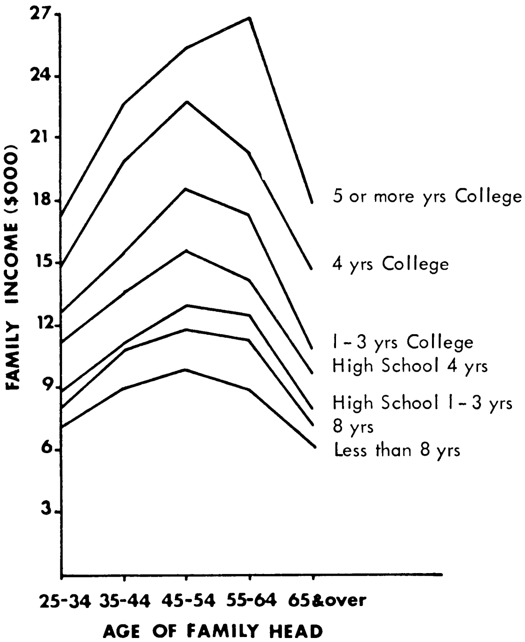

FIGURE 2A. AGE-EDUCATION PROFILES

Source: CPR, no. 90, pp. 79–81.

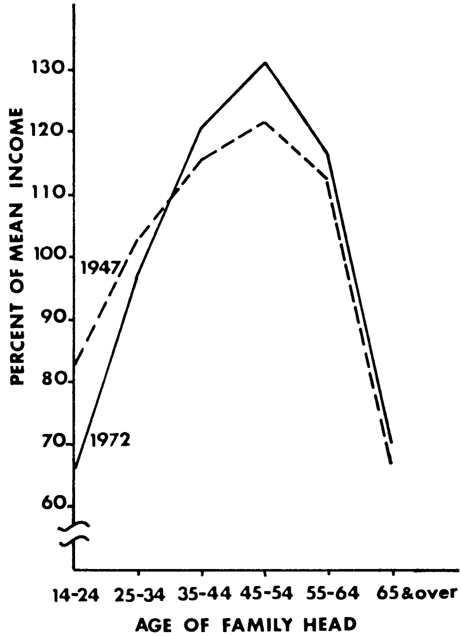

FIGURE 2B. INCOME PROFILE INDEXES

Source: Trends in the Income of Families and Persons in the United States: 1947–1964, p. 187; *CPR*, no. 90.

flat age-income profile.[1] While income would vary over the life cycle as indicated by curve AB, every family at a given stage of its life cycle would have exactly the same income as others at that stage—that is, all would have the same age-income profile. (We measure the stages of the life cycle by the age of the family head.) Now, taking the *U.S.* family population—its age distribution and average age-income profile—what would such a distribution of income look like on a Lorenz diagram? This is of course our new P-reference line of equality. We can derive it by taking average family income in each age group and ranking these groups by their mean incomes, remembering that every member of a group is assumed to have the same income. As expected, the young families and the over 65 age group rank at the bottom of the income scale. By using the percentage each group represents in the total population of families, a Lorenz-type curve was constructed from data in Table 1 and shown in Figure 1B as the P-reference line.[2] The hatched area between the 45 degree line and the P curve can be measured by a Gini ratio which is hereafter designated as the age-Gini. A standard Lorenz curve of family income (L) was then added in Figure 1B. The new area of inequality between the P curve and the L curve can also be measured by a Gini ratio which will be referred to as the Paglin-Gini, while the traditional Gini ratio based on the 45 degree line will be called the Lorenz-Gini. The Lorenz-Gini

[1] Equal lifetime incomes are specified in a no growth economy; with growth, the definition requires equal lifetime incomes only for members of the same age group or generation. This point is discussed below.

[2] A cubic spline function (cited in the Appendix) was fitted to the cumulative data points to derive the P curve.

TABLE 1

Age Group (1)	Mean Income (2)	Number of Families (thousands) (3)	Income of Group (millions) (4)	Cumulative Percent Families (5)	Cumulative Percent Income (6)
14–24	$ 7,892	4,194	$ 33,099	7.7	4.8
65 and Over	8,356	7,590	63,422	21.6	14.1
25–34	11,699	11,941	139,698	43.6	34.1
55–64	13,757	8,677	119,232	59.6	51.8
35–44	14,394	10,723	154,347	79.3	74.3
45–54	15,690	11,258	176,638	100.0	100.0
Totals		54,373	686,436		

Note: The P-reference curve is a Lorenz-type curve drawn up as illustrated above for 1972 family incomes (*CPR*, Series P-60, no. 90, pp. 51–53), with age groups ranked by mean incomes, not by ages.

minus the age-Gini equals the Paglin-Gini.[3]

Comparing the P and L curves in Figure 1B, and the Gini ratios of the areas shown, one is struck by the fact that fully one-third of the total area of inequality falls between the 45 degree line and the P curve. This shows what a large amount of ballast is included in the Lorenz-Gini measure, for clearly the age-income profile and the age structure of the population are not related to the long-run or lifetime degree of inequality in the economic system. In any case, the question of the optimum age-income profile is a different issue from that of equality as commonly conceived; our technique allows these two questions to be considered separately. Looking at the P curve, it is evident that rigidly egalitarian assumptions (i.e., equal lifetime incomes) may nevertheless produce a sizable area of inequality when annual income data are used. Hence, the Lorenz-Gini ratio (.359) is 50 percent greater than the Paglin-Gini (.239) which more closely approximates a measure of long-run interfamily inequality.

II. Traditional Attempts to Improve the Lorenz-Gini Measure

For some years it has been realized that the use of annual income data exaggerates the area of inequality. The suggested remedy has been the generation of hypothetical lifetime incomes since actual lifetime income data do not exist. But even if we had the data (historical and projected)

[3] In graphic terms, the Gini ratio is the Lorenzian area of inequality divided by the total area of the lower right triangle (Figure 1B). Algebraically, it is another measure of variance: $G=\Delta_1/2u$ where u is the mean income and

$$\Delta_1 = \frac{1}{N(N-1)}\sum_{j=1}^{n}\sum_{k=1}^{n}\{\,|x_j - x_k|\,f_jf_k\}$$

with x representing the values of the variate and f the frequencies (see G. U. Yule and M. G. Kendall, p. 146). Δ_1 is Gini's coefficient of mean difference; it is the summation of the absolute differences between each value of the variate and every other divided by the number of such pairings. The Paglin-Gini is simply: $P.G.=(\Delta_1-\Delta_1')/2u$ where Δ_1' equals the mean difference of the P curve distribution, namely, the hypothetical equality distribution in which all family incomes lie on curve AB. The Gini coefficient of mean difference and the concentration ratio are comparable to the more familiar measures of dispersion σ and $100\,\sigma/u$, and are monotonically related to them. Referring to Figure 1A, the Lorenz-Gini coefficient may be compared to the total variance (σ^2) around the mean, CD. The hatched area in Figure 1B (age-Gini) is conceptually comparable to the variance of AB from CD, while the Paglin-Gini coefficient measures the variation of values from AB. Education produces a greater arching of AB and thereby increases Lorenzian inequality which is measured from CD, but not the Paglin-Gini which is based on AB. The critique of the Lorenz-Gini measure developed here also applies to most other measures of income variation insofar as they use a single reference point such as the mean.

to construct lifetime incomes by consuming units, and converted these to constant dollars, the resulting Lorenz curve would show far more, rather than less, inequality. For in a dynamic economy with annual growth in real per capita income of 2 percent, there will be very large differences in lifetime incomes of older workers and young workers entering the labor force. With an average spread of 45 years between the new entrants and the retiring group, we can expect the former will have lifetime incomes well over 100 percent greater than the latter, and there is no practical redistribution scheme which would enable the older workers to approach the probable lifetime incomes of the younger. Also, given the upward trend in the labor-force participation rate of women, lifetime family incomes will increase even faster than the incomes of family heads. In a stationary economy, lifetime income equality is a workable hypothesis; in a growth economy, it becomes an almost unattainable goal. Therefore, to suggest that we retain the Lorenz 45 degree line but define it in terms of *lifetime* income equality gets us further from reality. While it eliminates the specification of a flat age-income profile, it saddles us with another unreasonable requirement: intergenerational lifetime income equality. This postulate, added to the empirical unreality of lifetime income data, reduces interest in this approach.

In contrast, the new P-reference function avoids these cumbersome problems. It recognizes that the past is irretrievably past, and that older generations cannot catch up with their younger contemporaries. It defines perfect equality at any point in time as equal incomes for all families at the same stage of their life cycle, but not necessarily equal incomes between different age groups. Annual income differences between age groups are solely a function of the average age-income profile; this reflects society's need for varying income over the life cycle as well as other basic facts relating to productivity, investment in human resources, and the work-leisure preferences of households, but only in an average way insofar as these factors express themselves through the age variable. The maintenance of P-reference equality posits that all members of a generation (or narrow age group) would have the same lifetime incomes; differences in lifetime incomes between generations would be mainly attributable to secular growth in real income, and such differences would be compatible with P-reference equality.

A second approach to the deficiencies of the Lorenz-Gini coefficient involves the use of age-specific Gini. The empirical coefficients available are not really specific by age of family head but in fact represent broad age groups. This introduces spurious income variance by not fully eliminating the effect of the age-income profile. However, even if we had truly age-specific Gini, we would have the problem of weighting and combining fifty-some measures into one coefficient. The use of the P curve is simpler, and allows us to graphically separate intrafamily and interfamily inequality; it also provides us with a new standard for evaluating quintile shares.

III. The Question of Consuming Units: What Should be Made Equal?

In organizing data for a Lorenz curve, the question of defining the appropriate consuming unit is usually considered a minor technical problem and hence glossed over by economists who are not specialists in this area.[4] However, the question, "What units should be made equal under

[4] One need only refer to our leading introductory economics text (Paul Samuelson, p. 85) which after nine editions muddies the issue by using a Lorenz diagram labelled "per cent of people" to represent tabular data showing "per cent families" as the consuming unit, thus overstating the area of inequality by 13 percent.

perfect equality?" has substantial effects on the area of inequality, second only to redefining the equality line itself. In Table 2, the Lorenz-Gini and Paglin-Gini concentration ratios are shown for three alternative consuming units. All are based on 1972 census data, but the Lorenz-Gini coefficients vary by as much as 29 percent, for example, between "households" and "persons in families." At one extreme, the use of households (which include single person units) assumes that equality is realized if all households (no matter what their size) have the same income; at the other extreme, equality is defined in terms of persons in families, which assumes that children are equivalent to adults and that there are no economies of scale in family units. When we use persons in families instead of number of family units, the inequality coefficient decreases because family size is positively correlated with income, not inversely as commonly thought.[5] The lower income families tend to be clustered more heavily at both ends of the age-income profile where family size is below average.

While the Lorenz-Gini measures vary by 29 percent, the difference increases to 92 percent when we include the Paglin-Gini ratios in the comparison. Two factors account for this tremendous range (.208 to .400): 1) the alternative definitions of the equality line in terms of a flat or curved age-income profile, and 2) the question of the appropriate consuming unit—households, families, or persons? Arguments have already been given for rejecting the gross Lorenz-Gini ratios in the first column of Table 2. Of the Paglin-Gini ratios in the third column, the household figure is the least acceptable since it lumps single person units along with the larger con-

TABLE 2—U.S. 1972 INEQUALITY COEFFICIENTS

Consuming Unit	Lorenz-Gini	Age-Gini	Paglin-Gini
Households: includes single person units and incomes of nonfamily members living in unit.	.400	.151	.249
Families: two or more persons (excludes unrelated persons in living unit).	.359	.120	.239
Persons in families.	.320	.112	.208

Sources: For Households, *CPR*, no. 89; for Families, *CPR*, no. 90; for Persons in families, *CPR*, no. 90, Table 2, and *Family Composition*, p. 55 for mean size of families, by age of family head. (All income data used in this article are before taxes, and represent money incomes only.)

suming units. This leaves us with the values for "families" (.239) and "persons in families" (.208) as defining a reasonable range for net interfamily inequality in the United States. These figures are a far cry from the usually published estimates of .35 to .40 and illustrate how easily the equality issue can be overblown. A very substantial part of the traditional area of inequality (one-third to one-half) is simply a function of the diversity in the ages and size of families, and the lifetime income pattern typical of a technically advanced society. Such inequality does not represent a limitation on lifetime opportunities, nor is it a quintessential evil to be obliterated if our society is to be considered just and humane.

IV. A Reassessment of the Trend of Inequality: 1947–1972

One major error resulting from the use of the Lorenz-Gini ratio has been the widely accepted conclusion that there has been no significant reduction of inequality from 1947 to 1972 despite the massive spending on education and training programs, the more generous cash and merit good transfers, and the legislative and judicial actions directed at bringing minor-

[5] *Current Population Reports*, P-60, no. 90, Table 1, p. 27. Family size increases over most of the income range, namely from $3,000 to $50,000.

TABLE 3—GINI CONCENTRATION RATIOS AND SHARE OF LOWEST QUINTILE: 1947–72

Year (1)	Lorenz-Gini (2)	Age-Gini (3)	Paglin-Gini (4)	Actual Share of Lowest Fifth (5)	Equality Share Using 20 Percent Point of P Curve (6)	Actual Share Divided by P-Equality Share (7)
1947	.378	.075	.303	5.1	15.5	.329
1948	.369	.076	.293	5.0	14.8	.338
1949	.379	.076	.303	4.5	15.1	.298
1950	.375	.077	.298	4.5	14.9	.302
1951	.361	.072	.289	4.9	14.8	.331
1952	.374	.078	.296	4.9	14.8	.331
1953	.360	*n.a.*	*n.a.*	4.7	*n.a.*	*n.a.*
1954	.373	.084	.289	4.5	14.6	.308
1955	.366	.088	.278	4.8	14.5	.331
1956	.355	.083	.272	4.9	14.3	.343
1957	.351	.089	.262	5.0	13.7	.365
1958	.354	.093	.261	5.1	13.5	.378
1959	.366	.090	.276	5.1	13.6	.375
1960	.369	.091	.278	4.9	13.8	.355
1961	.376	.090	.286	4.8	14.4	.333
1962	.365	.100	.265	5.1	13.5	.378
1963	.360	.098	.262	5.1	13.6	.375
1964	.352	.096	.256	5.2	13.7	.380
1965	.357	*n.a.*	*n.a.*	5.3	*n.a.*	*n.a.*
1966	.348	*n.a.*	*n.a.*	5.5	*n.a.*	*n.a.*
1967	.355	.110	.245	5.4	13.0	.415
1968	.343	.110	.233	5.7	12.9	.442
1969	.348	.115	.233	5.6	12.8	.438
1970	.355	.114	.241	5.5	12.9	.426
1971	.356	.119	.237	5.5	12.7	.433
1972	.359	.120	.239	5.4	12.9	.419

Sources: Column (2), years 1947–64, *Trends in the Income of Families and Persons in the United States: 1947–1964*, pp. 182–87; Gini ratios for years 1965–72 were computed by the cubic-spline method from data on family income in *CPR*, Series P-60. (See Appendix for a description of the method used.) Column (3) lists the Gini ratios of the area between the 45 degree line and the P-reference curve computed each year from age-income data in same sources as Column (2). Column (4) equals (2) minus (3) and represents net interfamily inequality. Columns (5), (6), and (7) used the same data sources and P curves as columns (2) and (3).

Note: n.a.= Not available because median rather than mean incomes by age groups were given for those years.

ities and underprivileged groups into the mainstream of the economy. A fresh analysis of the income data using the P-reference curve indicates a considerable reduction of net inequality and a marked improvement in the share of the lowest quintile. Table 3 shows the historical data on inequality coefficients. Column (2) lists the standard Lorenz-Gini concentration ratios with only a slight decline in inequality evident. This forms the basis of the current view that inequality has not changed in the last twenty-five years. Column (3) lists the age-Gini ratios; these must be computed for each year since the age-income profile and the age structure change slowly over time. We may note a significant increase in the age-Gini from 1947 to 1972; this is related to the expansion of higher education which results in a greater arching of the average age-income profile (Figure 2), and to the increase in the percent of the aged and young adults in the population.[6]

[6] In 1947 young and old families (head below 25 years and over 65) together equalled 13.9 percent of all families; in 1972 they were 21.7 percent of the total.

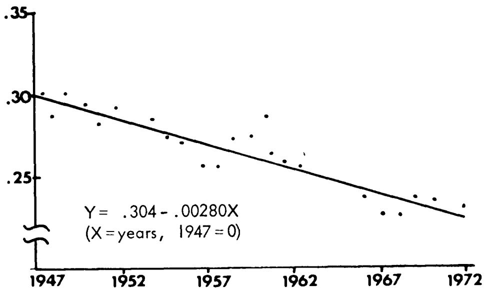

FIGURE 3A. TREND OF PAGLIN-GINI RATIOS

Note: $r = .94$; $t = 12$.

Source for both figures: Table 3.

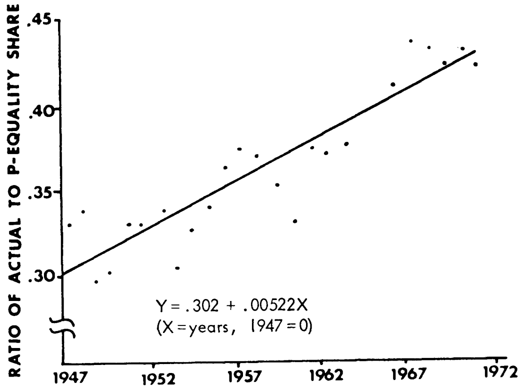

FIGURE 3B. TREND IN INCOME SHARE OF LOWEST QUINTILE USING P-REFERENCE CURVE

Note: $r = .91$; $t = 10$.

Column (4) is the net measure of inequality expressed by the Paglin-Gini ratio, and is equal to column (2) minus (3). It reveals the decline in interfamily inequality of income, unobscured by changes in the age-income profile and in the age composition of the population. The least squares trend line fitted to column (4) data is shown in Figure 3A. In contrast to the traditional view, the equation indicates that inequality has declined 23 percent in the 25-year period, 1947–72.

V. Reevaluation of the Lowest Quintile

Let us consider the effect of the new equality reference line on the relative position of the lowest fifth of the income recipients. Using the traditional Lorenz curve (Table 4), the poorest families (in 1972) have 5.4 percent of the income pie whereas in a perfectly egalitarian society they would appear to get 20 percent of the income; hence it is implied that a measure of their deprivation is shown by the fact that they receive only 5.4/20 or 27 percent of the income share which would accrue to them under conditions of perfect equality. This grossly exaggerates the income deprivation of the lowest fifth. Using the more realistic definition of perfect equality embodied in our P curve, the relative position of the lowest quintile changes significantly, for we see that even under conditions of perfect equality of lifetime incomes (given the current age distribution and age-income profile), there would still be a "poorest" 20 percent of the families having only 12.9 percent of the income pie. Now it is appropriate to compare the 5.4 percent with the 12.9 percent, rather than with the 20 percent figure, unless we are prepared to argue that equality of lifetime income is not a reasonable norm, and that flat annual income from teenage to retirement and death is also somehow a necessary condition of equality. By employing the P-reference curve, we see that the poorest families receive 5.4/12.9 or 41.9 percent of the income which would accrue to them in an egalitarian utopia (Table 4). Hence, a revision of the equality norm has resulted in a 55 percent improvement in the relative assessment of the lowest income group, namely from 27 to 42 percent of the equality income share. (A similar analysis using the P curve as the standard results in a reduction of the share of the top

TABLE 4—1972 FAMILY INCOMES: LORENZ AND P CURVE EQUALITY COMPARISONS

Quintiles	Percent of 1972 Income (Actual)	Perfect Equality Lorenz Shares	Perfect Equality Shares Using P Curve	Actual Income Share as a Percent of: Lorenz Share	P-Curve Share
Lowest	5.4	20	12.9	27.0	41.9
2	11.9	20	17.8	59.5	66.9
3	17.5	20	21.5	87.5	81.4
4	23.9	20	22.9	119.5	104.4
Highest	41.3	20	24.9	206.5	165.9

Source: CPR, Series P-60, no. 90. P-equality shares interpolated from cubic-spline curve fitted to P-curve data points.

quintile from 2.07 times to 1.66 times the equality share, see Table 4.)

We may now begin to understand why the poorest 20 percent historically seem to be so resistant to improvements in terms of the conventional Lorenz curve analysis: to a large degree, the low percentage of the income pie going to this group is a built-in result of the age-income profile coupled with the age distribution of the population, and is not purely related to the condition of a permanent class of people excluded from the average level of real income enjoyed by most families. This has profound implications, for the failure to understand the nature and extent of economic inequality has led to a distorted assessment of our economic system.

This distortion has been further amplified by a statistical decision to exclude need-based in-kind transfers from the definition of income. As a matter of social policy, we have decided to mitigate poverty by making large transfers in the form of public housing, rent supplements, food stamps and food assistance, medicaid, and social services such as day care, etc. We then blithely exclude these transfers from the statistics on poverty and inequality and wonder about the lack of improvement in the share allotted to the lowest quintile! Fifteen years ago, in-kind transfers were small and could be overlooked, but now they exceed the cash allotments and are growing at a much faster rate. In fiscal year 1973, expenditures on the major income-tested transfer programs (federal, state, and local) totaled \$30.7 billion of which 55 percent or \$16.9 billion were in-kind transfers. While the cashing out of in-kind transfers is a complex task which will be dealt with in a subsequent article, a loose estimate of their distributive impact can be made. Since we are focusing on family units, only 80 percent of the in-kind transfers or \$13.5 billion is applicable. Now assume that 20 percent of the \$13.5 billion goes for administration. This leaves \$10.8 billion as an addition to the income of the bottom quintile, boosting its 1972 income share from 5.4 to 7.0 percent. Hence if in-kind transfers are included as income, the lowest quintile has about 54 percent of the P curve equality share of 12.9 percent.[7]

Aside from the sharp increase of in-kind

[7] The data on transfer programs are derived from *The Budget of the U.S. Government, FY 1975, Appendix*, pp. 203–05, 440–42, 494, and 808. Programs such as medicare, which affect many low income families, have been excluded from the totals because they are not income tested. The 80 percent figure represents the proportion of low income persons living in families (*CPR*, no. 98, p. 13). The 20 percent for administrative costs is a rough estimate which may be low for existing programs but would be too high if a cash transfer strategy (negative income tax) were used.

transfers which characterized the period 1947–72, the historical position of the lowest quintile purely in terms of money income has improved significantly. While this improvement has been obscured by the crude Lorenz approach (Table 3, column (5)), it is revealed by comparing in each year the actual share received by the lowest quintile with the P curve equality share. These ratios are shown in Table 3, column (7), and are plotted in Figure 3B along with the least squares trend. We may conclude that the overall reduction of inequality in the period 1947–72 has been accompanied by an upward trend in the money income share of the lowest quintile, namely, from 33 to 42 percent of the equality share.

VI. Wealth Inequality

From income inequality let us move on to the distribution of wealth. Our criticisms with some modification also apply to the Lorenz-Gini wealth coefficient. Simply stated, the problem is this: the 45 degree line of equality requires that wealth holdings among families be equal regardless of the age of the head of household. Given a typical age distribution of the population, this means that cumulative annual saving, plus interest compounded, can have no effect on individual family wealth. Again, this is an added specification which most persons would consider an unnecessary and unrealistic constraint even in an egalitarian society. It would seem sufficient to define equality as a social condition in

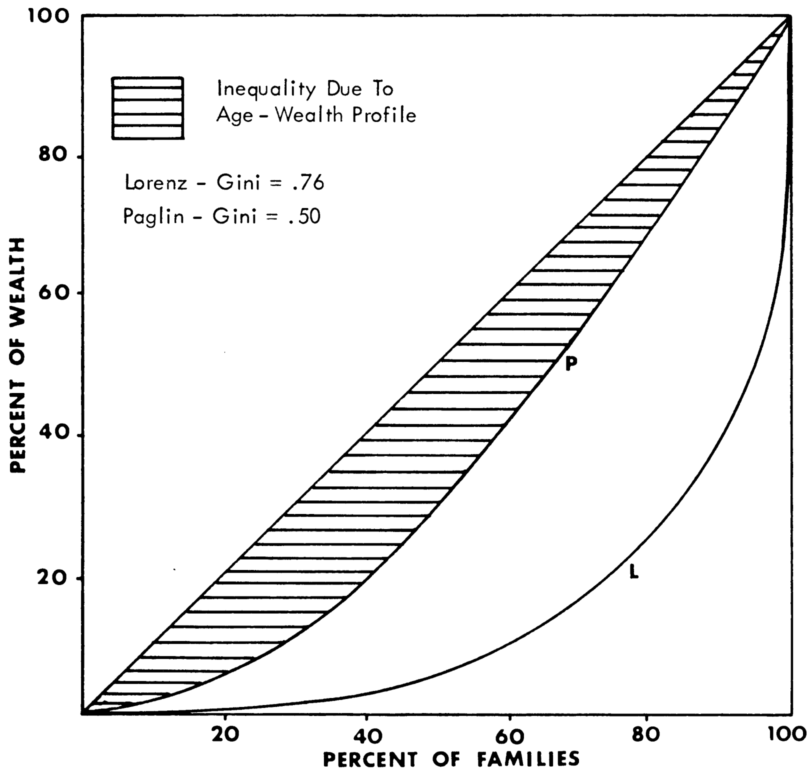

FIGURE 4. *U.S.* DISTRIBUTION OF WEALTH, 1962

Source: Projector and Weiss, pp. 110, 151.

which inheritance of wealth is not a significant factor in the *relative* distribution of wealth, all lifetime incomes (for a given generation) are equal, and family wealth is a function mainly of saving and time, with everyone having the same rate of saving (or dissaving) at a given stage of the life cycle. These assumptions would produce equality of wealth for families in the same age bracket, but would allow differences of wealth based on age. With this definition, what would a P-reference function for the United States look like, using the average age-wealth profile derived from Federal Reserve Board data (see Dorothy Projector and Gertrude Weiss)? The results are shown in Figure 4. The area between the 45 degree line and the P-reference function is hatched, and the Lorenz curve of wealth is also shown. It is apparent that a significant percentage of the total area of inequality (about one-third of it) would exist even in a rigidly egalitarian society, and therefore, this segment of wealth inequality reflects the age structure and the social savings function rather than fundamental (lifetime) inequality in the economic system. For the United States (in 1962) the Lorenz-Gini of wealth was .76 compared with a Paglin-Gini of .50; hence, the traditional measure has overstated the degree of interfamily inequality of wealth by about 52 percent.

VII. Summary and Conclusion

For over sixty years we have employed a measure of inequality—the Lorenz curve—based on an unrealistic standard of equality. When used with annual income data, Lorenzian equality requires a flat age-income profile as well as equality of lifetime incomes. While the use of lifetime income data would negate the overspecification of equality built into the 45 degree line, empirical and theoretical considerations make this approach impractical. By developing a new equality reference function, suitable for use with annual income data, we not only avoid cumbersome data problems, but we open up to explicit examination the meaning of perfect equality. We also separate the issue of *intra*family variation in income over the life cycle (accounting for one-third of Lorenzian inequality) from the more basic issue of *inter*family differences in lifetime incomes. An application of the new concepts to *U.S.* income and wealth data reveals that estimates of inequality have been overstated by 50 percent, and the trend of inequality from 1947 to 1972 has declined by 23 percent. On equity as well as efficiency grounds there is a substantial case for rejecting the Lorenzian standard of absolute equality. The overstatement of inequality has lent false urgency to the demand for rectification of our income distribution.

Appendix

The traditional method of calculating the Gini concentration ratio involves a straight line approximation between plotted points of the Lorenz curve, and a simple formula for determining the resulting area (see James Morgan, Appendix). This, however, underestimates the area of inequality, especially if the number of data points drops below eight, a common occurrence since we frequently have income shares by quintiles. There seemed to be two obvious remedies but both were found wanting: fit an appropriate curve type by least squares; this however didn't pass through all the data points and was therefore rejected. The second approach involved fitting an $n-1$ degree polynomial which passed through every data point but showed unpredictable flips between the points. What is wanted is a smooth, continuous function which will pass through all the points, whether few or many, and be usable for interpolation between the points. A cubic-spline function described in Brice Carnahan et al., p. 63, fulfilled these conditions, and was used to derive L curves and P-reference curves for all the years required. A computer program was written to generate about 40 interpolated points between every two actual data points, and by integration provide the area of inequality and the Gini ratio.

As a test of the program, we had the benchmark Gini ratios for 1947–64 (Census, *Trends in the Income of Families . . .*) computed from disaggregated data. Using quintile shares plus the share of the top 5 percent for these 18 years, Gini ratios were computed by both the straight line and the cubic-spline methods, and compared with the Census coefficients. The straight line method produced underestimations averaging 3.5 percent. The cubic-spline method yielded correct estimates (to three significant figures) for one-third of the years, and for the entire sample produced an average error of only 0.6 percent (.002 Gini point). The smooth curves and the interpolations provided by the program also made possible meaningful comparisons of various points on the P and the L curves.

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